OPTIMIZING THE DESIGN OF THE SIX-PORT JUNCTION

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Abstract

A set of optimum design objectives for the Six-Port technique is derived from an examination of the system equations. The optimization criterion is to minimize the sensitivity of the solution to calibration errors. The resulting set of design specifications gives significant improvement in error cancellation.

Introduction

The value of the Six-Port technique as an alternative to conventional heterodyne methods for measuring microwave network parameters has been shown in recent investigations [1,2,3,4,5,6,8,9]. Many hardware realizations of the Six-Port Junctions have been proposed in the literature for use as microwave network analyzers [1,3,8,9,10]. The ability to measure both magnitude and phase without the use of a phase locked source makes the Six-Port Analyzer very attractive. However, this advantage is somewhat diminished by the requirement that calibration constants be provided for each frequency of interest [7,8,9]. Thus while the source need not be phase locked, its frequency must be known to some accuracy, or the Six-Port based network analyzer must contain frequency measurement capability. In addition, memory must be provided to store these constants within the instrument. These limitations greatly reduce the advantages of the Six-Port in swept frequency measurements where the frequency may change rapidly over a wide range.

The limitation comes about from the high sensitivity of the solution to small changes in the hardware constants. These constants (coupling coefficients, phase shifts, etc.), will change slightly as the frequency is swept over the Six-Port's operating range. Thus the higher the sensitivity to small changes in these constants, the larger the number of frequencies at which the Six-Port must be calibrated, and the more accurately the frequency must be known or measured. Alternatively, unit to unit differences due to fabrication errors may also alter the calibration and thus system performance.

Solution of the Arbitrary Six-Port

Thus, in order to optimize the Six-Port junction for use as a network analyzer, it is desirable to investigate the sensitivity of the solution to small changes in the design constants. Engen [4] has expressed the design constants of an arbitrary Six-Port conveniently as

$$P_1=|A_1a+B_1b|^2=K_1|a|^2|\rho-q_1|^2 \quad (1)$$

$$P_2=|A_2a+B_2b|^2=K_2|a|^2|\rho-q_2|^2 \quad (2)$$

$$P_3=|A_3a+B_3b|^2=K_3|a|^2|\rho-q_3|^2 \quad (3)$$

$$P_4=|A_4a+B_4b|^2=K_4|a|^2|\rho-q_4|^2 \quad (4)$$

where $q_i = -A_i/B_i$, $K_i = |B_i|^2$, and $\rho = b/a$.

Any linear Six-Port with four ports terminated in power meters, figure 1, can be represented in the form of (1) - (4).

The significance of the value of the q's is best demonstrated by example. If q is equal to 0, the equation describes a directional coupler coupled only to the incident wave. A value of q equal to unity describes equal coupling to both incident and reflected waves, while the phase of q determines the phase difference. The K's in (1) - (4) are constants of proportionality and are hereafter assumed to be incorporated in P1 through P4. In this representation the basic hardware structure is described by four complex constants. The solution of ρ involves solving a system of four nonlinear simultaneous equations in three unknowns ($\mathrm{Re}(\rho)$, $\mathrm{Im}(\rho)$, $|a|$). Thus the system is overdetermined. It is this extra degree of freedom which allows significant improvement in desensitivity to perturbations in the values of the q's. The optimization problem is then to choose values for the q's which minimize the sensitivity of the solution to small changes in the q values around nominal design values.

To this end, the solution of the complex reflection coefficient is expressed as the intersection of circles in the ρ plane [4]. These circles are the locus of points satisfying the measurement data and are found by dividing pairs of equations (1)-(4). For example equation (1) divided by equation (2) yields

$$\left(\rho - \frac{q_1P_2-q_2P_1}{P_2-P_1}\right)^2 = \frac{P_1P_2}{(P_2-P_1)^2}|q_1-q_2|^2 \quad (5)$$

In general, dividing P_L by P_M gives a circle having a radius R

$$R_{LM}^2 = r_{LM}\frac{P_L\,P_M}{(P_M-P_L)^2}|q_L-q_M|^2 \quad (6)$$

and a center

$$C_{LM} = \frac{q_LP_M - q_MP_L}{P_M-P_L} \quad (7)$$

There are six such circles, all having a common intersection inside the unit circle of the plane or Smith Chart. This intersection is the value of the complex reflection coefficient ρ [4].

The six circles are a set of six equations in two unknowns (having eliminated $|a|$). There can only be three independent equations so we must choose three of the six. The basis of this choice will be determined after investigating the sensitivity of the solutions.

Optimization of the Six-Port

When small errors are introduced into the values of the q's, the circles no longer intersect in a common point, and errors are introduced in the final result. The sensitivity of the final result to changes in the values of the q's will depend on the algorithm used to average the intersections. In order to remain as general as possible at this point, it is necessary to investigate the sensitivities independently of such an algorithm. By examining the sensitivity of the radii and centers of the individual circles, and not the intersections, the problem of specifying an algorithm is bypassed.

The sensitivity of each circle to q variation can be expressed as the partial derivatives of the radii and the centers with respect to the real and imaginary part of the q values. Thus for the circle formed by dividing PL by PM the following derivatives may be written:

$$\frac{\partial r_{LM}}{\partial \alpha_L} = \frac{(2\alpha_L - 2\mathrm{Re}\,\rho)P_M|q_M-q_L|^2 + P_L P_M(2\alpha_L - 2\alpha_M)}{(P_M-P_L)^2} - \frac{4(\alpha_L - \mathrm{Re}\,\rho)P_L P_M |q_M-q_L|^2}{(P_M-P_L)^3} \qquad (8)$$

$$\frac{\partial r_{LM}}{\partial \beta_L} = \frac{(2\beta_L - 2\mathrm{Im}\,\rho)P_M|q_M-q_L|^2 + P_L P_M(2\beta_L - 2\beta_M)}{(P_M-P_L)^2} - \frac{4(\beta_L - \mathrm{Im}\,\rho)P_L P_M |q_M-q_L|^2}{(P_M-P_L)^3} \qquad (9)$$

$$\frac{\partial \mathrm{Re}C_{LM}}{\partial \alpha_L} = \frac{P_M - \alpha_M(2\alpha_L - 2\mathrm{Re}\,\rho)}{P_M-P_L} + \frac{(\alpha_L P_M - \alpha_M P_L)(2\alpha_L - 2\mathrm{Re}\,\rho)}{(P_M-P_L)^2} \qquad (10)$$

$$\frac{\partial \mathrm{Re}C_{LM}}{\partial \beta_L} = \frac{-2\alpha_M(\beta_L - \mathrm{Im}\,\rho)}{P_M-P_L} + \frac{2(\beta_L - \mathrm{Im}\,\rho)(\alpha_L P_M - \alpha_M P_L)}{(P_M-P_L)^2} \qquad (11)$$

$$\frac{\partial \mathrm{Im}C_{LM}}{\partial \alpha_L} = \frac{-2\beta_M(\alpha_L - \mathrm{Re}\,\rho)}{P_M-P_L} + \frac{2(\alpha_L - \mathrm{Re}\,\rho)(\beta_L P_M - \beta_M P_L)}{(P_M-P_L)^2} \qquad (12)$$

$$\frac{\partial \mathrm{Im}C_{LM}}{\partial \beta_L} = \frac{P_M - 2\beta_M(\beta - \mathrm{Im}\,\rho)}{P_M-P_L} + \frac{2(\beta_L - \mathrm{Im}\,\rho)(\beta_L P_M - \beta_M P_L)}{(P_M-P_L)^2} \qquad (13)$$

where $\alpha_i = \mathrm{Re}(q_i)$ and $\beta_i = \mathrm{Im}(q_i)$

It is desired to choose values for the q's which will minimize (8) - (13). When either q_L or q_M are zero or infinity, each of the partials (8) - (13) go to zero. Engen [4] has chosen infinity as one value for his realizations. However, while this is indeed insensitive to perterbations in q, it is also insensitive to the value of ρ, and thus contains no information about ρ. Therefore it appears that equal to zero is a better choice.

Equations (8) - (13) provide an additional insight; as P_L approaches P_M, the sensitivities approach infinity. Thus in choosing which set of three circles from (7) - (8) to use in solving for ρ, one must not allow P_L and P_M to be equal. This can be avoided for q_L equal to 0 if $|q_M| > \sqrt{2}$. On the other hand, if q_M is too large, the dynamic range of P_1 becomes small, and the measurement contains little information about ρ [4]. Thus the remaining three values of q's should have magnitudes very nearly equal to $\sqrt{2}$. Finally, as noted by Engen [4], they should be dispersed symetrically about the orgin, ideally at 120° displacements, however it does not appear critical that they be exactly at 120° increments.

Realizations

To test the above theory, a narrow band test jiis, figure (2), has been built. By adjusting the sliding section of line, or trombone, any desired phase of q can be simulated. The magnitude of q is fixed at $\sqrt{2}$ by the 3 dB pad. A q of zero is obtained by measuring directly at the directional coupler's reflected port. The variation in the q's with frequency for this system is shown in figure (3), while a typical error in phase measurement is shown in figure (4). Similar measurements using q1 equal to infinity were unable to provide reliable phase information at frequencies more the one percent from the center frequency. It should be noted that while these errors seem large, significant improvement in performance would be obtained with a broad band system. Thus many of the presently used Six-Ports may be further improved by simply modifying one of the sampling ports to sample the reflected wave rather than the incident wave. For example the circuit of figure (5) is a rearrangement of the circuit proposed by Engen which can be implimented over very broad frequency ranges [3].

Conclusions

From the above development it is seen that use of the set of design constants outlined makes the Six-Port optimally insensitive to errors in the calibration constants. This added stability against calibration errors may be used to increase the frequency range over which a given calibration vector may be used. Alternatively, the added stability could be used to allow an increase in fabrication tolerances for a given Six-Port realization.

References

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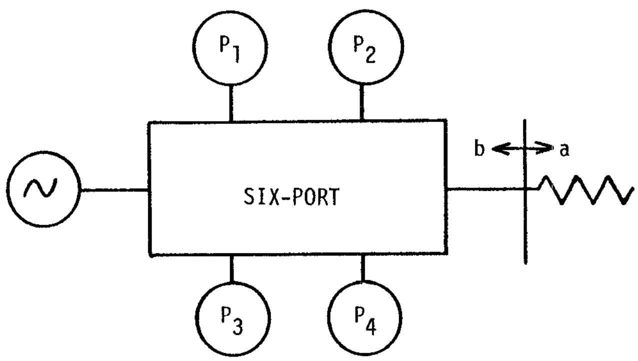

FIGURE 1: Arbitrary Six-Port Junction.

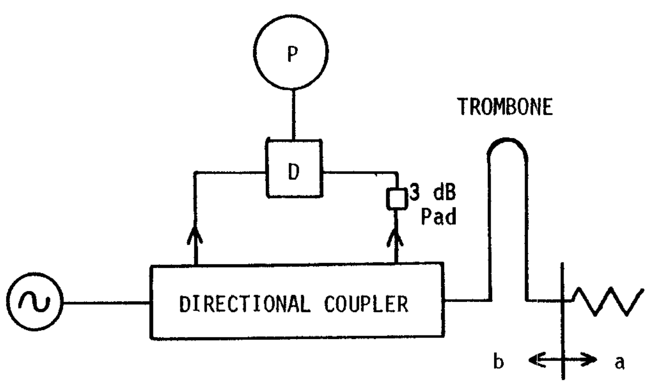

FIGURE 2: Test Jig for Six-Port Realization. D is a power divider/combiner.

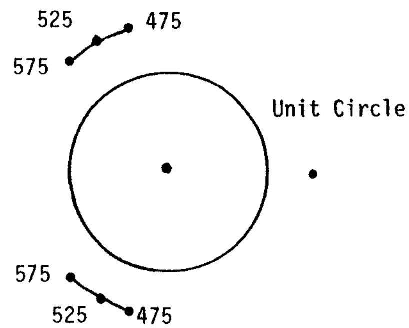

FIGURE 3: Variation of q's for the circuit of Fig. 2. Frequency is in MHz.

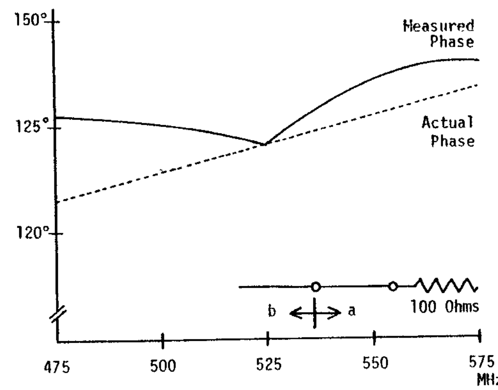

FIGURE 4: Typical Phase error using circuit of Fig. 2. $|\rho|=0.33$.

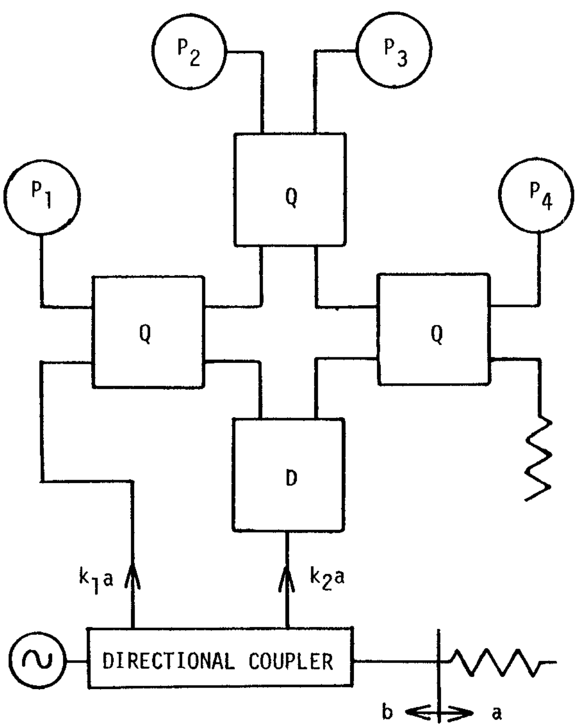

FIGURE 5: Proposed Six-Port. D is a power divider, Q is a quadriture hybrid.